Differential Rotation of Individual Sunspots and Pores

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ABSTRACT

The analysis of the rotation rate of individual sunspots and pores was performed according to the data of the processing of observations by the Solar Dynamics Observatory/Helioseismic and Magnetic Imager (SDO/HMI) in the period 2010–2024. Sunspots stood out in the images in the continuum. To accurately track the spots, we processed 5 images for each day. To determine the polarity of the magnetic field, we superimposed the contours of sunspots on observations of magnetic fields at the same time. This made it possible to track the movement of more than 210 thousand individual sunspots and pores. It is found that the rotation rate is influenced by the rotation rate of the solar atmosphere and the systematic proper motions of the spots. Sunspots and pores of the leading polarity have a rate of meridional movement $\approx 2.4\%$ faster than spots of the trailing polarity. We also found that regular sunspots, which have umbra and penumbra, rotate $\approx 1.5\%$ faster than solar pores, in which penumbra is absent. The dependence of the rotation rate on these area is found. For sunspots with an area of $S > 10 \ \mu$ hm, the rotation rate is practically independent of these area. Small sunspots, with an area of lower than $S < 10 \ \mu$ hm, rotate $\approx 1.7\%$ slower.

Keywords: Sunspots(1653) — Solar differential rotation(1996) — Bipolar sunspot groups(156)

1. INTRODUCTION

The rotation rate of the Sun around its axis varies from many parameters. Different methods and different tracers used to track movement give different results (Beck 2000). Long-term measurements of the rotation rate of the upper layers of the solar atmosphere by averaging over longitude and time show that the rotation rate depends on latitude and depth (Howe et al. 2000). Sunspots are often used to study long-term rotation variations. Sunspots rotate at a different rate than non-magnetic plasma. The most available data for studying long-term variations in the rotation of the Sun are data on sunspot groups. Based on the analysis of Mount Wilson observations from 1921 to 1982 (Howard et al. 1984), the law of rotation of sunspot groups $\omega(\theta) = 14.52 - 2.84 \sin^2\theta \text{ deg/day}$ was found. According to the processing of the Greenwich Photoheliographic Results 1874-1976 (Balthasar et al. 1986), the law of rotation was found $\omega(\theta) = 14.557 - 2.85 \sin^2\theta$

In addition to latitude, the rotation rate of sunspots is influenced by other factors. A number of authors have found that the rotation rate decreases with the area of the sunspot groups (Howard et al. 1984; Gupta et al. 1999; Nagovitsyn et al. 2018). Rotation depends on the type of sunspot groups (Balthasar et al. 1986): C-groups exhibit high rate, and H- and J-spots are the smallest. Some authors note that the that the rotation of young sunspot groups occurs about 7% faster than the rotation of older sunspot group (Tuominen 1962; Balthasar et al. 1982; Pulkkinen & Tuominen 1998). On the other hand, (Javaraiah & Gokhale 1997; Sivaraman et al. 2003) found that long-lived sunspot groups accelerate these rotation over time. In (Kutsenko et al. 2023) the authors found that the active region can exhibit both acceleration and deceleration during occurrence, whereas during attenuation, the rotation rate remains virtually unchanged. There are variations in different cycles and phases within cycles, as shown by (Balthasar et al. 1986) and (Howard et al. 1984). The highest rotation rates are observed during the phases of minimum activity.

There are much fewer studies based on the analysis of the movement of individual sunspots. Newton & Nunn (1951), studying the movements single unipolar recurrent sunspots found the dependence of the sidereal velocity of rotations of the sine of latitude: $\omega(\theta) = A + B \cdot \sin^2 \theta = 14.38 - 2.38 \sin^2 \theta \text{ deg/day}$. In (Kutsenko 2021) the rotation rate was found for unipolar sunspots $\omega(\theta) = 14.28 - 2.4 \sin^2 \theta - 2.24 \sin^4 \theta \text{ deg/day}$. In (Howard et al. 1984) it was found that sunspot groups rotate about 1% slower than individual sunspots. In (Permata & Herdiwijaya 2019) the average rotational

velocity $\omega(\theta) = 14.376 + 0.6 \sin^2 \theta \, \text{deg/day}$ was found. As the authors point out, the positive constant at $\sin^2 \theta$ may be associated with errors in determining the rate.

Gilman & Howard (1984, 1985) performed the analysis of the rotation rate variation of individual sunspots according to the Mount Wilson Observatory. They processed photographic plates of observations in "white" light for the period 1917–1983. In (Gilman & Howard 1985) sunspots with a lifetime of at least 2 days were taken for analysis. It was found that the leading sunspots rotate faster than the following sunspots, by about 0.1 deg/day, or 14 m/s.

It should be taken into account that the measured rotation rate may be superimposed by sunspots own movements. In this study, we will look at the rate of longitude movement of individual sunspots, focusing on different types of sunspots. Section 2 presents a description of the data, a method for identifying sunspots and determining the velocity of movement. In Section 3.1, the velocity of longitude movement of sunspots of various types such as pores and regular sunspots is investigated. In section 3.2, the velocities for sunspots of the leading and trailing polarity are considered. Section 4 presents a discussion of the results obtained.

2. DATA AND METHOD

For the analysis, we used Solar Dynamics Observatory/Helioseismic and Magnetic Imager (SDO/HMI), with a cadence of 45 seconds of the hmi.Ic_45s and hdmi.M_45s series made at the same time. We took 5 images for each day at times close to 00:00, 05:00, 10:00, 15:00 and 20:00 UT (http://jsoc.stanford.edu/ajax/exportdata.html). The main data for processing were images in "white" light (hmi.Ic_45s), on which sunspots and sunspot umbra were highlighted. The observation data were processed automatically. To identify sunspots and pores, we used a sunspot detection procedure similar to that used at the Kislovodsk Mountain Astronomical Station (KMAS) to identify sunspots for synoptic observations. The analysis of KMAS synoptic observations uses white light observations made with a 150 mm diameter photoheliograph. To extract sunspots, automatic sunspot segmentation procedures are used. But also an important feature in the processing program in Kislovodsk is the option where the operator can change methods, method parameters and perform object segmentation manually. This provides greater flexibility in the use of the program, which is especially important when processing observations with defects and in clouds (Tlatov et al. 2014b,a).

Initially, the darkening towards the edge of the Sun was eliminated in the images of the Sun in the continuum, and the level of quiet Sun I_{qs} was determined. To eliminate limb darkening and determine the background intensity, the solar disk was divided into 12 segments by polar angle. In each segment, the dependence of intensity on radius was found. The intensity at each point depending on the polar angle and distance was found by interpolation (Tlatov et al. 2014a). Then, regions of existence of sunspot groups of with a size of $\pm 6^{\circ}$ in latitude and $\pm 8^{\circ}$ degrees in longitude from the center of the sunspot groups were highlighted on the disk. The coordinates of the sunspot groups were taken from the data http://solarcyclescience.com/activeregions.html.

A sunspot search was conducted in these regions. To search for the photosphere-penumbra boundary $I_{\rm pu}$, the method of "growing" from a "seed" with a minimum intensity of $I_{\rm mn} < 0.92I_{\rm qs}$ was used. The growing procedure was carried out in increments of $10^{-2}I_{\rm qs}$. The area gain was calculated at each step. When reaching a level close to $I_{\rm qs}$, the increase became large. The procedure was then stopped and the second to last step level was applied as the $I_{\rm pu}$ threshold. Umbra and penumbra were allocated for each sunspot. The procedure for searching for the umbra-penumbra boundary $I_{\rm up}$ was similar, but instead of the $I_{\rm qs}$ intensity, the intensity of the penumbra-photosphere boundary $I_{\rm pu}$ was used. The search started with points with an intensity lower than $0.92I_{\rm pu}$. If the penumbra could not be allocated, then such spots were marked as solar pores.

The technique of sunspot detection is also described in (Tlatov et al. 2014b; Tlatov 2022a). To determine the polarity of the magnetic field, we superimposed the boundaries of sunspots on the observations of magnetic fields (hdmi.M_45s) and determined the maximum magnetic field along the line of sight B_{LOS} . Pores in which the intensity of the magnetic field was $B_{\text{LOS}} < 100$ G were excluded from consideration (Tlatov 2022a). In total, more than 87 thousand sunspots and more than 200 thousand pores were allocated in the period 01.05.2010-31.05.2024.

The data of individual sunspots based on the results of processing ground-based observations in the period 2010–2024 are available in tabular form //solarstation.ru/archive and in graphic form on the website https://observethesun.com. Detailed differences between the characteristics of pores and regular sunspots are presented (Tlatov & Pevtsov 2014; Tlatov et al. 2014a; Tlatov 2022a).

Figure 1 shows the average monthly values of the area according to the KMAS ground-based synoptic observation processing data and the SDO/HMI automatic processing data used by us for analysis. The correlation coefficient was

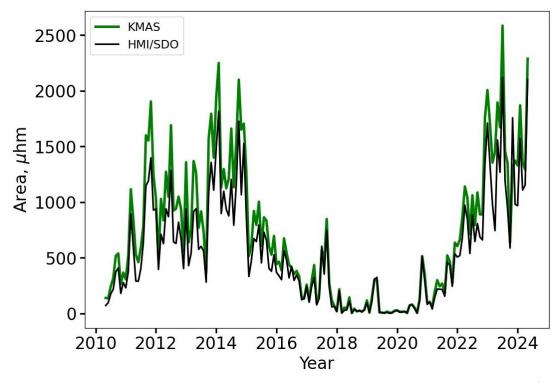


Figure 1. Comparison of monthly sunspot area data from KMAS synoptic ground observations and SDO/HMI automatic processing data.

r=0.98. Some differences in the data are mainly due to the difficulty of determining the area at the limb during the automatic procedure. Despite the significant number of pores in the total area, they account for approximately 15%.

To determine the rotation rate, you need to track sunspots on time-adjacent images. To do this, we compared sunspots using these characteristics, primarily coordinates, the sign of the polarity of the magnetic field and the area. We searched for pairs of spots at various points in time at the minimum expected distance, taking into account the polarity of the magnetic field and the limitation on the difference in area. In total, more than 210 thousand pairs were allocated in neighboring images of the Sun with a difference of 5 hours. The sequence determination procedure applied to determining the lifetime of sunspots and pores is presented in (Tlatov 2023). Next, we determined the rate of movement of the geometric center of sunspots and pores $\omega = \Delta \phi / \Delta t$, where $\Delta \phi$ is the longitude shift, Δt is the time interval.

3. ANALYSIS RESULTS

3.1. Rotation of Sunspots and Solar Pores

Sunspots can be divided into two types according to these morphology. Regular sunspots that have umbra and penumbra, and pores (in which the penumbra cannot be distinguished). Pores have a much smaller area than regular sunspots. Pores form sunspot groups of class A and B according to the Zurich classification (Tlatov et al. 2014a; Tlatova et al. 2022). In (Tlatov & Pevtsov 2014; Tlatov 2023) it was shown that the magnetic fields of pores and regular sunspots have different dependences on the area. This may indicate different physical processes in the formation of these types of spots. Therefore, it is advisable to study the rotation of these two types of spots separately.

To study the dependence of rate on latitude, we looked for average rate values in latitude intervals of 5° degrees. Figure 2 shows the sidereal rotation rate of pores and sunspots. Using the least squares method, we obtained the following dependencies for sunspots: $\omega_{sp}(\theta) = 14.56 - 3.094 \sin^2 \theta - 0.04 \sin^4 \theta$, and for pores: $\omega_{pr}(\theta) = 14.34 - 2.097 \sin^2 \theta - 0.01 \sin^4 \theta \text{ deg/day}$.

Sunspots rotate faster than solar pores and have a greater degree of latitude differentiation. The average pore area for which the rotation velocity was found was $S_{\rm pr} \approx 9.4 \ \mu {\rm hm}$, and the average sunspot area $S_{\rm sp} \approx 133.6 \ \mu {\rm hm}$. Thus, larger sunspot objects rotate faster than pores.

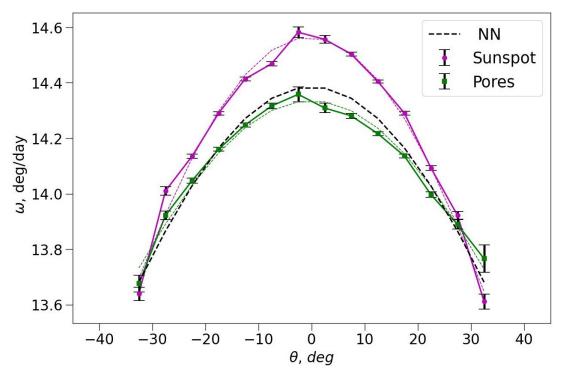


Figure 2. The rotation rate of individual sunspots and pores. The black dotted line (NN) represents the rotation rate of unipolar sunspots (Newton & Nunn 1951). A confidence interval for approximations is presented. The dotted lines represent approximations for different types of spots.

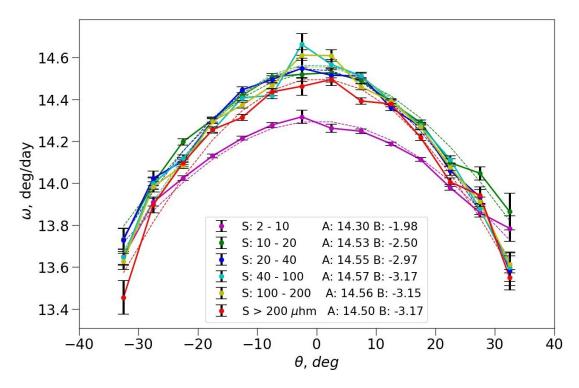


Figure 3. Rotation rate of sunspots of different area intervals. The dotted lines represent approximations for spots of different areas. The coefficients of approximations $\omega(\theta) = A + B \cdot \sin^2 \theta$ are presented.

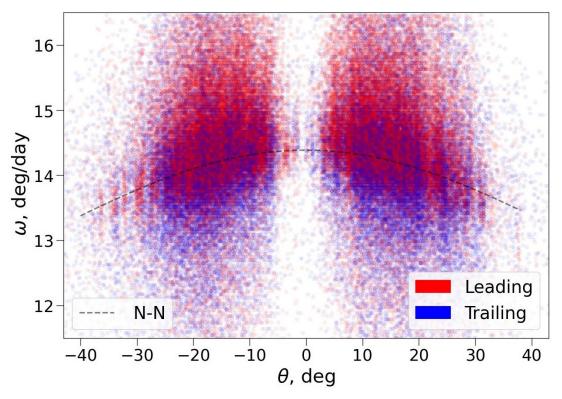


Figure 4. The scattering diagram of longitudinal velocity of leading and trailing polarity sunspots.

3.2. Rotation of Sunspots of Different Area

In this section, we will consider the change in the rotation rate from the area of sunspots and pores. There are contradictory results on the issue of changing the rotation rate from the area. To study the question of the rotation rate from the area, we divided all sunspots into area intervals $S : 2-10, 10-20, 20-40, 40-100, 100-200, >200 \mu$ hm. For this analysis, we did not separate sunspots according to morphology, that is, into pores, sunspots without penumbra and regular sunspots. And only the area was taken into account.

Figure 3 shows graphs dependence of the rotation rate on latitude. The coefficients of approximation of coefficients A and B are also presented. It can be noted that, depending on the area, all the spots can be divided into two groups. Small area spots $S < 10 \ \mu$ hm, which rotate relatively slowly and their rotation velocity at the equator is 14.30 deg/day. For larger area spots $S > 10 \ \mu$ hm, the rotation velocity at the equator is $\approx 14.56 \ deg/day$ and changes slightly with area. The areas of the spots $S < 10 \ \mu$ hm usually correspond to the pores. Figure 3 confirms the results of Figure 2.

3.3. Rotation of Sunspots of Leading and Trailing Polarity

Another sample of sunspots of various types may be the separation by the polarity of the magnetic field. To determine the polarity of the field, we used the superposition of the boundaries of sunspots identified in the continuum on magnetograms taken at the same time. The Hale's law was used to determine the polarity sign of the leading and trailing sunspots.

In total, ≈ 117 thousand spots of the leading polarity and ≈ 95 thousand of the trailing polarity were identified, in which the velocity of longitude movement was determined. Figure 4 shows a scattering diagram of the velocity of longitude movement of regular sunspots of different polarities. Leading polarity sunspots are shown in red, trailing polarity sunspots are shown in blue. We can note that the graph shows the separation of the velocity of movement for these types of spots

Figure 5 shows approximations of the displacement velocity for sunspots and pores of the leading and trailing polarities. For sunspots of the leading polarity, the rotation rate can be approximated by the formula: $\omega_{\rm ld}(\theta) = 14.574 - 2.225 \sin^2\theta - 0.02 \sin^4\theta \, \deg/\text{day}$. For trailing polarity spots: $\omega_{\rm tr}(\theta) = 14.216 - 2.60 \sin^2\theta - 0.01 \sin^4\theta \, \deg/\text{day}$. Sunspots of the leading polarity rotate $\approx 2.4\%$ faster than trailing polarity sunspots.

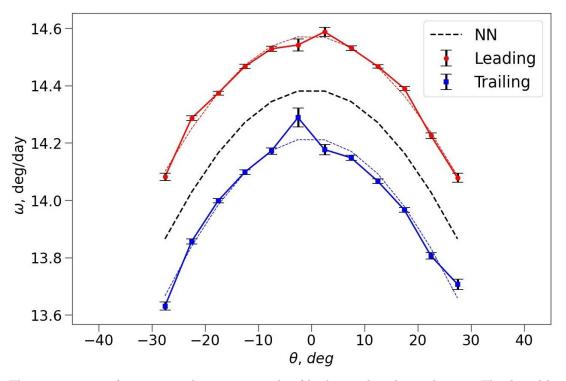


Figure 5. The rotation rate of sunspots and pores separately of leading and trailing polarities. The dotted lines represent approximations for spots of different polarity.

Figures 6 shows the dependences of the rotation rate of the leading and trailing polarity spots separately for different types of spots: pores and sunspots. For pores, this dependence can be represented as: $\omega_{pr}^{ld}(\theta) = 14.50 - 1.71 \sin^2\theta \, \text{deg/day}$ for leading polarity; and $\omega_{pr}^{tr}(\theta) = 14.17 - 2.28 \sin^2\theta \, \text{deg/day}$ for trailing polarity. For sunspots, the dependencies are as follows: $\omega_{sp}^{ld}(\theta) = 14.65 - 2.85 \sin^2\theta \, \text{deg/day}$ leading polarity; and $\omega_{sp}^{tr}(\theta) = 14.34 - 3.48 \sin^2\theta \, \text{deg/day}$ for trailing polarity.

Again, we can note that the pore rotation rate is lower than the rotation rate of sunspots, both for spots of the leading and trailing polarity.

Near the equator in Figures 5, 6 there are deviations from the patterns. This is due to errors in determining the leading polarity in the hemispheres. It is possible that due to random processes, sunspots and pores cross the equator and thereby violate Halle's law of polarity for the hemispheres. We took such sunspots to be spots of leading or trailing polarity. But at the same time, these rotation rate corresponded to sunspots from the opposite hemisphere.

The dependence of the differential rotation rate of sunspots of different magnetic polarity on the area is shown in Figure 7. Here, spots near the equator $\theta < \pm 15^{\circ}$ are considered. The following can be noted. Small sunspots with an area of up to $S < 10 \ \mu$ hm rotate more slowly than spots with a larger area. For sunspots with an area of S:10-50 μ hm, the rotation rate practically does not change with area. For sunspots of leading polarity, with increasing area at $S > 50 \ \mu$ hm, the rotation rate decreases. But for spots of trailing polarity, the dependence of the rate on the area is not visible.

4. DISCUSSION

We performed the analysis of the longitude movement of individual sunspots. We found that the velocity of movement of sunspots significantly depends on these type. Table 1 shows the values of approximations of rotation rates from the area and for various types of sunspots. The separation of sunspots into pores, that is, spots without penumbra, and regular sunspots with penumbra, showed that regular sunspots rotate $\approx 1.5\%$ faster than pores ($A_{\rm pr} = 14.34$, $A_{\rm sp} = 14.56 \text{ deg/day}$). At the same time, the degree of differential rotation of sunspots is $\approx 47\%$ higher than for pores ($B_{\rm pr} = 2.1, B_{\rm sp} = 3.09 \text{ deg/day}$) (Figure 2). This result is confirmed by the analysis of the rotation rate of sunspots of various areas (Figure 3). Small spots $S < 10 \ \mu$ hm rotate slowly.

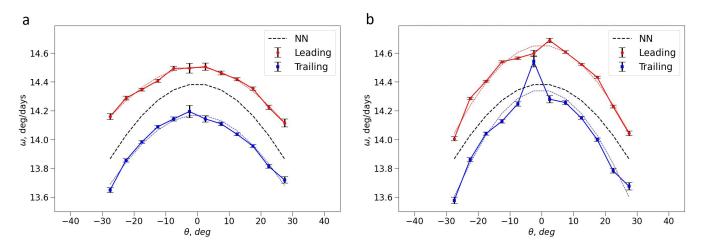


Figure 6. The same as Figure 5, but separately for pores (left) and sunspots (right).

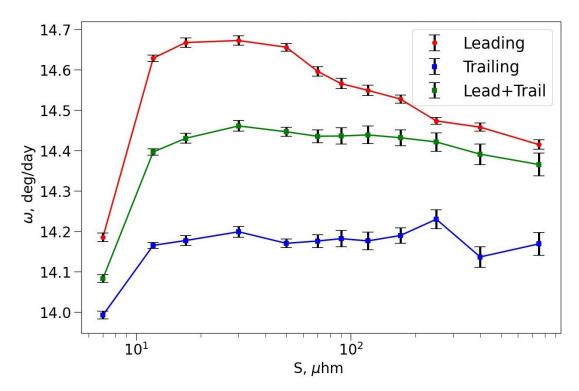


Figure 7. Rotation rate of spots of different magnetic polarity depending on the area for spots with latitude in the range $\theta < \pm 15^{\circ}$.

The rotation rate f sunspots with an area of $S > 10 \ \mu$ hm is practically independent of the area (Figure 3). This contradicts the conclusions of previous studies (Howard et al. 1984; Gupta et al. 1999; Nagovitsyn et al. 2018) for the rotation of sunspot groups. It is possible that there are errors in determining the rotation rate of sunspot groups. Consider measuring the rate of sunspot groups near the limbs, where measurements are difficult due to the effects of projection and the Wilson effect (Wilson & Maskelyne 1774). On the eastern limb, the average coordinate of the groups is shifted towards the leading sunspot during measurements. On the western limb, the average measured longitude of the sunspot group shifts to the trailing sunspots. This means that the longitude distance increases during measurements, which leads to errors in measuring rotation rate. The greater the longitude of the group, the greater the error may be. Since the longitude and size are interrelated, this leads to an error in determining the rotation rate,

Pores	
all	$\omega_{pr}(\theta) = 14.34 - 2.01 \mathrm{sin}^2 \theta$
Leading	$\omega_{pr}^{ld}(\theta) = 14.50 - 1.71 \mathrm{sin}^2 \theta$
Trailing	$\omega_{pr}^{tr}(\theta) = 14.17 - 2.286 \mathrm{sin}^2 \theta$
	Sunspots
all	$\omega_{sp}(\theta) = 14.56 - 3.01 \mathrm{sin}^2 \theta$
Leading	$\omega_{sp}^{ld}(\theta) = 14.65 - 2.85 \mathrm{sin}^2 \theta$
Trailing	$\omega_{sp}^{tr}(\theta) = 14.34 - 3.48 \mathrm{sin}^2\theta$
Area	
$S<10~\mu{\rm hm}$	$\omega(\theta) = 14.30 - 1.98 \mathrm{sin}^2 \theta$
$10-20~\mu{\rm hm}$	$\omega(\theta) = 14.53 - 2.50 \mathrm{sin}^2 \theta$
$20-40~\mu{\rm hm}$	$\omega(\theta) = 14.55 - 2.97 \mathrm{sin}^2 \theta$
$40-100~\mu{\rm hm}$	$\omega(\theta) = 14.57 - 3.17 \mathrm{sin}^2 \theta$
$100-200~\mu{\rm hm}$	$\omega(\theta) = 14.56 - 3.15 \mathrm{sin}^2 \theta$
$S > 200 \ \mu \text{hm}$	$\omega(\theta) = 14.51 - 3.17 \mathrm{sin}^2 \theta$

Table 1. Summary of sunspots and pores sidereal rotation rates (deg/day).

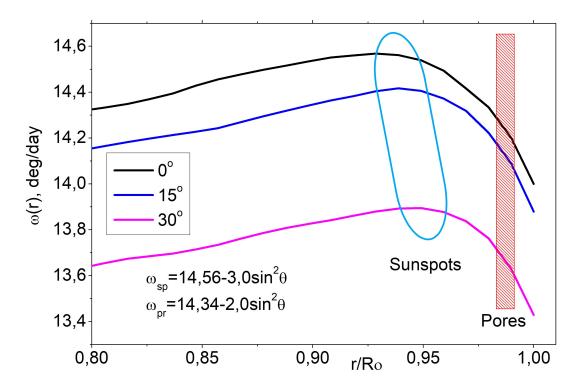


Figure 8. Scheme of formation of sunspots and pores at different depths for latitudes 0° , 15° , 30° inside the leptocline $(r \gtrsim 0.95 R_{\odot})$. Rotation data are taken from (Howe et al. 2000).

which increases with the area of the group. Another factor influencing velocity measurement errors may be the faster decay of trailing regions (Petrovay 1993).

The difference in the rotation rate of different types of sunspots is often explained within the framework of the anchor hypothesis. This hypothesis assumes that the magnetic flux beam is fixed at some depth in the convection zone (Gilman & Howard 1984; Rhodes et al. 1990; Kutsenko et al. 2023). As a result, the rotation rate on the surface is determined by the rotation rate of the plasma at the depth of fixation. It is known that in the upper layer of the solar convective zone, called the leptocline ($r > 0.95R_{\odot}$), the rotation rate changes rapidly with depth (Howe et al. 2000).

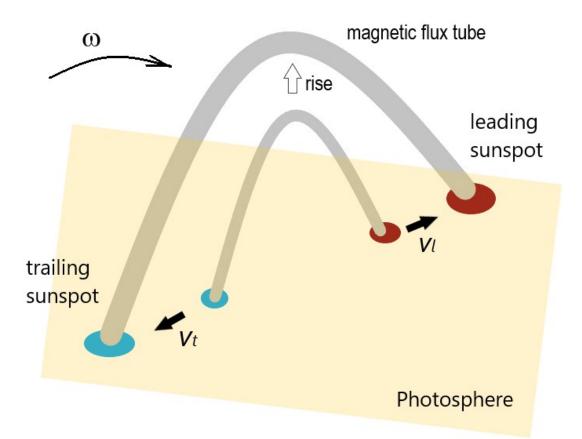


Figure 9. A simplified scheme of the possible formation of an additional velocity of the longitudinal displacement of sunspots during the ascent of a magnetic flux tube.

At depths of $r > 0.95 R_{\odot}$, the rotation rate is maximum. Pores have a much smaller area than regular sunspots. In our study, the average pore area for which the rotational rate was found in neighboring images was $S_{\rm pr} \approx 10 \ \mu$ hm, or diameter $d_{\rm pr} \approx 1.954 S^{0.5} \approx 6$ Mm, and the average sunspot area $S_{\rm sp} \approx 133 \ \mu$ hm, or $d_{\rm sp} \approx 20$ Mm. A comparison of the rotational velocities of different types of spots found in this study (Figure 2) with the results of the rotation rate in depth (Howe et al. 2000) shows that the rotation of the pores is close to the region of delayed rotation in the leptocline at a depth of $h_{\rm pr} \approx 0.98 R_{\odot} \approx 14$ Mm. The rotation rate of regular sunspots is close to the values of the maximum rotation at a depth of $h_{\rm sp} \approx 0.95 R_{\odot} \approx 35$ Mm (Figure 8). It is possible that sunspots and pores are predominantly formed with a ratio of diameter to depth of the spot $h/d \approx 2$. Since the region of maximum rotation varies with depth (Figure 8), it is possible that the degree of differential rotation for regular sunspots is therefore higher than for pores.

The second result of our study indicates that the velocity of the longitudinal movement of sunspots depends on the polarity of the magnetic field. To separate the spots, we used the polarity of the magnetic field. A similar result was obtained earlier (Gilman & Howard 1985). To separate sunspots, the authors employed the leading the leading and trailing sunspots of a sunspot groups when processing the plates in white-light. Sunspots and pores of the leading polarity have rotation rate of $\approx 2.4\%$, or 0.26 deg/day, or 36 m/s faster than the trailing polarity spots (Figures 5). Such a difference in velocities may be due to additional factors affecting the horizontal movement of the spots. Sunspots are visible as the exits of the arch of magnetic flux tubes on the photosphere. At the initial stage of ascent, the distance between sunspots is lower than in the later stages, since the distance between the legs of the arches fixed on the photosphere increases with distance from its top. This leads to the appearance of an additional observable velocity $v_{\rm l}$ for the leading spot directed along the rotation. For the trailing spots of the part, the $v_{\rm t}$ velocity is directed against rotation, which creates the effect of a slow rotation rate (Figure 9). It is likely that the rate of ascent and therefore the rate of longitudinal flow differs for pores and regular sunspots. In Figure 6, we see that the velocity differences are greater for pores than for regular sunspots. This corresponds to the hypothesis of the ascent of the flux

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tube. At the initial stage of the ascent, the apparent velocity of the run-up of the bases of the arches of the magnetic flux tube will be greater, as can be seen from Figure 6.

The hypothesis of the influence of the ascent of the magnetic flux tube on the velocity of longitudinal flows corresponds to anomalies in the rotation rate near the equator (Figure 6). If, due to turbulence, the trailing or leading part of the bipolar group, one hemisphere, enters the other hemisphere, then we will mistakenly consider them leading or trailing according to Hale's law. But these rate will differ from the pattern for this hemisphere.

Studies of the longitudinal movement of individual sunspots and pores can help in determining the distribution of rotation rate with depth and can be used in models of dynamos and the formation of magnetic field flux tubes.

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